CALCULATION OF RADIATIVE-CONVECTIVE RECUPERATORS

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Linearization of the boundary conditions on the partition surfaces is used to obtain simple design relationships for the determination of the heating surface of radiative-convective recuperators.

New trends in engineering development make it necessary to devise calculation methods for recuperators operating at high temperatures. Such recuperators are, in particular, essential parts of apparatuses for the direct conversion of thermal to electrical energy [1–4] and also find application in the chemical, power, metallurgical, and forging industries [5–8].

The main difficulty in obtaining design relationships for such recuperators is that the radiative component of the heat flux imparts significant nonlinearity to the initial differential equations. Hence, it is natural to attempt to linearize these equations. The simplest approach to this consists in the introduction of a heat transfer coefficient $\alpha = \alpha_R + \alpha_C$, composed of convective and radiative components [9, 10]. The drawback of the method is that α_R changes very considerably with temperature along the apparatus and, hence, the choice of an appropriate numerical value for this quantity is very approximate.

Another approach—the method of successive approximations, described in [7]—is very awkward and laborious.

Good results are given by the computer calculations used by the authors of [5, 16], but the use of machine techniques entails the writing of programs for each variant of the boundary conditions which determine the operation of the apparatus, and, hence, this method is very time-consuming.

We will consider a radiative-convective recuperator (Fig. 1). We will assume that the two heat-exchanging gases are diathermal and the casing of the apparatus is thermally insulated, so that no heat is lost to the surroundings. The hot gas, flowing in the annular space 2, transmits heat by convection through the partition 3 to the gas flowing through tube 1. The wall of the casing is also heated by the hot gas and transmits heat to the wall of the inner tube by radiation.

It is known that in the case of purely convective heat transfer in a recuperator the heat transfer in any cross section of the apparatus is determined by the difference in the temperatures of the fluids in this section. This does not apply to radiative heat transfer between the casing wall and the inner tube, since the radiative heat flux in any section will depend not only on the difference of the fourth powers of the wall temperatures in it, but also on the heat fluxes due to radiative interaction with other points of the tube or casing, as shown by the arrows in Fig. 1.

Nevertheless, it is assumed in papers devoted to the calculation of radiative recuperators [5, 7] that the radiant flux is determined entirely by the difference $\vartheta^4 - \vartheta''^4_{12}$. In certain conditions this assumption is valid and ensures sufficient accuracy for engineering calculations. The only requirement is that the distance between the walls exchanging radiant heat should be small in comparison with the total length of the apparatus, since it is easy to show from purely geometric considerations that for a solid angle of 0.6 sr 90% of the total energy of hemispherical emission will correspond to a portion of the length of the heat-transfer surface equal approximately to four times the distance between the casing and the inner tube. If the change of the temperature of the fluids is small in this portion, which will be the case in the above-mentioned conditions, the convective and radiative heat exchange can be regarded as taking place in the same cross section and will be characterized by the temperatures of the fluids and partitions in it. McAdams [11] arrived at a similar conclusion from a qualitative consideration of the heating of articles in a long oven with variable wall temperature.

The heat exchange between the fluids in steady-state parallel flow in a recuperator is described by the following system of differential equations [12]:

$$T' = \vartheta'_{12} - \frac{W_1}{\alpha_1 s}\,\frac{dT'}{dx}, \tag{1}$$

$$T'' = \vartheta''_{12} \mp \frac{W_2}{\alpha_2 s}\,\frac{dT''}{dx}, \tag{2}$$

$$\frac{d^2\vartheta_{12}}{dz^2} = 0, \tag{3}$$

the last equation being applicable to the case of a cylindrical partition if the ratio of the outer and inner diameters of the tube is close to unity.

Equation (3) ignores the heat transfer along the partition, i.e., along the x axis, which does not introduce an appreciable error if the length of the apparatus is more than ten times the thickness of the partition [12]. Solution (3) must satisfy the following boundary conditions in any cross section of the apparatus:

$$-\lambda \left.\frac{d\vartheta_{12}}{dz}\right|_{z=-\delta/2} = \alpha_2 (T'' - \vartheta''_{12}) + \varepsilon\sigma_0 (\vartheta^4 - \vartheta''^4_{12}), \tag{4}$$

$$-\lambda \left.\frac{d\vartheta_{12}}{dz}\right|_{z=+\delta/2} = \alpha_1 (\vartheta'_{12} - T'). \tag{5}$$

We must also have another relationship

$$\alpha_2 (T'' - \vartheta) - \varepsilon\sigma_0 (\vartheta^4 - \vartheta_{12}^{''4}) = 0, \qquad (6)$$

derived from the condition of thermal equilibrium of the apparatus casing. Bearing in mind that the solution of (3) is

$$\vartheta_{12} = C_1 + C_2 z, \qquad (7)$$

and making it conform with conditions (4) and (5), we obtain after simple transformations

$$C_1 = \frac{1+\mathrm{Bi}_1}{1+2\mathrm{Bi}_1}\,\vartheta_{12}'' + \frac{\mathrm{Bi}_1}{1+2\mathrm{Bi}_1}\,T';\ \mathrm{Bi}_1 = \frac{\alpha_1\delta}{2\lambda}, \qquad (8)$$

$$\vartheta_{12}'' - \vartheta^* = \frac{\varepsilon\sigma_0\delta(1+2\mathrm{Bi}_1)}{2\lambda[\mathrm{Bi}_1 + \mathrm{Bi}_2(1+2\mathrm{Bi}_1)]}(\vartheta^4 - \vartheta_{12}^{''4}), \qquad (8a)$$

$$\vartheta_{12}' = \frac{1}{1+2\mathrm{Bi}_1}\,\vartheta_{12}'' + \frac{2\mathrm{Bi}_1}{1+2\mathrm{Bi}_1}\,T', \qquad (8b)$$

from which with reference to (6)

$$(T'' - \vartheta)/(\vartheta_{12}'' - \vartheta^*) = 1 + \mathrm{Bi}_1/\mathrm{Bi}_2(1+2\mathrm{Bi}_1) \equiv b, \qquad (9)$$

where

$$\vartheta^* = \frac{\mathrm{Bi}_1 T' + \mathrm{Bi}_2(1+2\mathrm{Bi}_1)T''}{\mathrm{Bi}_1 + \mathrm{Bi}_2(1+2\mathrm{Bi}_1)} \qquad (10)$$

is the temperature which would be established on the surface of the partition on the side of the hotter fluid in the absence of radiative heat transfer.

Determining ϑ from (9) and substituting its value in (8a), we ultimately find

$$\frac{\vartheta_{12}''}{\vartheta^*} - 1 = a\vartheta^{*3}\left\{\left[\frac{T''}{\vartheta^*} - b\left(\frac{\vartheta_{12}''}{\vartheta^*} - 1\right)\right]^4 - \left(\frac{\vartheta_{12}''}{\vartheta^*}\right)^4\right\}, \qquad (11)$$

where

$$a = \varepsilon\sigma_0(1+2\mathrm{Bi}_1)/[\alpha_1 + \alpha_2(1+2\mathrm{Bi}_1)]. \qquad (12)$$

Equation (11) establishes the relationship

$$T''/\vartheta^* = f(\vartheta_{12}''/\vartheta^*,\ a\vartheta^{*3},\ b). \qquad (13)$$

This functional relationship, as can be seen from (11), cannot be put in an explicit form suitable for use. In Fig. 2 this relationship is depicted graphically for a fairly wide range of arguments.

A point of interest is that for each fixed value of b the value of the ratio T''/ϑ^* depends practically entirely on $\vartheta_{12}''/\vartheta^*$, and the effect of parameter $a\vartheta^*$ over a very wide range of its variation is slight.

This circumstance indicates that combined radiative and convective heat transfer is determined not by the values of such characteristics as ε, α, λ, and ϑ^* taken separately, but by their combination in the form $a\vartheta^{*3}$. A change in this parameter when its value is small significantly affects the value of T''/ϑ^*, but when $a\vartheta^*$ attains a certain value this effect becomes insignificant, as is clearly illustrated in Fig. 3, which shows that, beginning at approximately 0.05, an increase in $a\vartheta^{*3}$ by a factor of ten or more alters the value of T''/ϑ^* by only a few per cent. The relative value of this change decreases with increase in the parameter b.

The condition $a\vartheta^{*3} \geq 0.05$ means in practice that the recommended method is applicable to recuperators with temperature $\vartheta^* > 600°$ K. In a recuperator, if we ignore the variation of the thermophysical characteristics of the gas with temperature, the variation of the parameter $a\vartheta^{*3}$ along the length of the apparatus is due entirely to the temperature ϑ^*, which depends in turn on the temperature of the fluids in the specified conditions of convective heat transfer. The value of ϑ^* in parallel-flow apparatus in many important practical cases increases or decreases by a factor of not more than two or three, although, as was shown in [12], it may even remain constant in certain conditions.

Thus, in a wide range of variation of the parameter relationship (13), as Fig. 2 shows, can be well approximated by a family of straight lines:

$$T''/\vartheta^* = A_1 + B_1\vartheta_{12}''/\vartheta^*$$

or

$$\vartheta_{12}'' = T''/B_1 - A_1\vartheta^*/B_1 = AT'' + B\vartheta^*. \qquad (14)$$

In a first approximation A and B are functions only of parameter b.

The values of coefficients A and B in Eq. (14) are

b	$A\cdot 10^2$	B	Maximum Error, %
2	31.0	0.685	4.0
4	20.8	0.790	2.5
10	8.00	0.919	1.0
25	2.60	0.974	<1

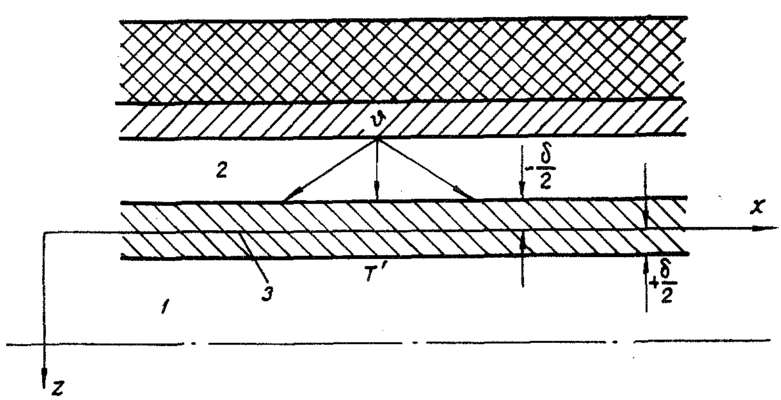

Fig. 1. Diagram of radiative-convective recuperator.

Substitution of the corresponding values from formulas (14), (8b), and (10) in (1) and (2) leads to the following two equations:

$$T' = \left[\frac{1-(A+B)}{k^*}\,\alpha_2 + 1\right] T'' \pm R_{21}\,\frac{dT''}{dv_x}, \qquad (15)$$

$$T'' = \frac{\alpha_1/k^* - \alpha_1/\alpha_2}{1+2\mathrm{Bi}_1 + A\alpha_1/k^*}\,T' + \frac{1+2\mathrm{Bi}_1}{1+2\mathrm{Bi}_1 + A\alpha_1/k^*}\,\frac{dT'}{dv_x}. \qquad (16)$$

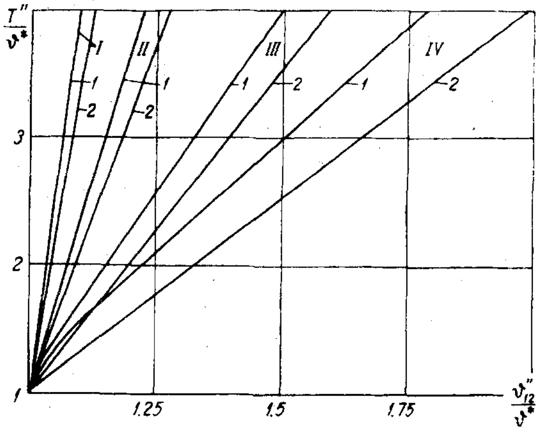

Fig. 2. T''/ϑ^* as a function of $\vartheta''_{12}/\vartheta^*$; 1) for $a\vartheta^{*3} = 0.05$; 2) $a\vartheta^{*3} \to \infty$; I) for B = 25; II) 10; III 4: IV) 2.

The first term in the square brackets of formula (15) is very much less than unity, as the table shows that A + B ≈ 1. Taking this into account with $\alpha_2/k^* < 20$, we obtain from (15) and (16)

$$\frac{d^2T'}{dv_x^2} + (1 \pm R_{12} + \xi)\,\frac{dT'}{dv_x} = 0. \qquad (17)$$

In formulas (15)–(17)

$$k^* = Bk,\ v_x = k^*F_x/W_1,\ \xi = (\alpha_1/k^*)\,(1-B)/(1+2\mathrm{Bi}_1). \qquad (18)$$

From an examination of (17) we can conclude that calculation of a radiative-convective heat-exchanger reduces to the solution of a differential equation describing the process in an apparatus of purely convective type if the ratio of the water equivalents is altered by a value ξ, and the heat transfer coefficient is multiplied by B.

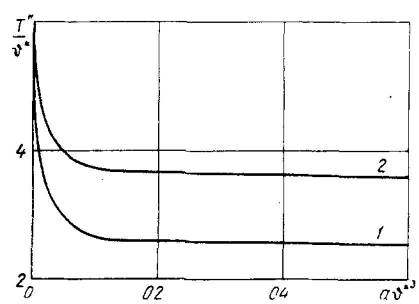

Fig. 3. T''/ϑ^* as a function of the parameter $a\vartheta^{*3}$ for b = 2 (1) and 4 (2).

For the two cases of flow of the fluids the solution of (17) has to satisfy the following boundary conditions: for parallel flow

$$T'|_{v_x=0} = T'_i,\ \frac{dT'}{dv_x}\bigg|_{v_x=0} = (1+\xi)\,(T''_i - T'_i), \qquad (19)$$

for counterflow

$$T'|_{v_x=0} = T'_i,\ \frac{dT'}{dv_x}\bigg|_{v_x=0} = (1+\xi)\,(T''_f - T'_i). \qquad (19a)$$

We finally obtain for parallel flow of the fluids

$$v \equiv k^*F/W_1 = -(1+R_{12}+\xi)^{-1}\ln\{1-\Theta'_f\,[1+R_{12}/(1+\xi)]\} \qquad (20)$$

and for counterflow

$$v = -(1-R_{12}+\xi)^{-1}\ln\{1-[\Theta'_f/(1-R_{12}\Theta'_f)]\,[1-R_{12}/(1+\xi)]\}, \qquad (21)$$

where

$$\Theta'_f = (T'_f - T'_i)/T''_i - T'_i). \qquad (22)$$

The presence of radiative heat transfer in the parallel-flow apparatus leads to greater utilization of the temperature head than in the case of purely convective heat transfer. The maximum attainable dimensionless temperature in this case is determined from (20) by putting the expression under the logarithm sign equal to zero; then

$$(\Theta'_f)_{\max} = (1+\xi)/(1+\xi+R_{12}).$$

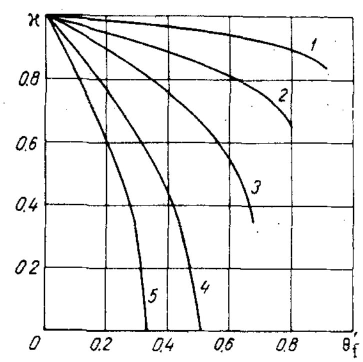

Fig. 4. Ratio $\varkappa$ of heat-transfer surfaces in radiative-convective recuperator with parallel flow and counterflow of fluids in relation to dimensionless temperature Θ'_f for $\xi = 1$ and ratio R_{12} of water equivalents equal to: 1) 0. 2; 2) 0. 5; 3) 1. 0; 4) 2. 0; 5) 3. 0.

It is of interest to compare the efficiencies of the two kinds of heat transfer (Fig. 4).

We note that parallel flow in the range of final fluid temperatures attainable with such flow is more efficient than counterflow. This result is very important, since it leads to the conclusion that radiative-convective recuperators should not be designed for counterflow of the fluids when the conditions of operation are such that $\Theta'_f < (\Theta'_f)_{\max}$.

The physical explanation of this circumstance is that in parallel flow the wall temperature is lower and the amount of radiant heat transmitted from the casing to the inner tube is greater than in counterflow.

The above design formulas were obtained, as indicated, for the case of heat exchange between two diathermal gases. It can be shown, however, that in certain conditions they can be applied with a reasonable degree of accuracy when the heat-exchanging gases are radiating media.

An analytical solution, involving several simplifying assumptions, of the problem of the heat exchange of a gray gas flowing in a tube with constant wall temperature was given in [13]. The use of this solution is difficult, since it includes a generalized thermal diffusivity of unknown numerical value.

An examination of the expression given by Nevskii [14] for the resultant heat flux between parallel gray surfaces separated by a space filled with an absorbing and reemitting medium,

$$E_{\mathrm{r}} = \frac{\sigma_0 (\vartheta^4 - \vartheta_{12}''^4)}{1/\varepsilon + 1/\varepsilon_{12} - 1 + \varepsilon''/(2 - \varepsilon'')},$$

shows that when the value of ε'' is small this expression is converted to the formula for radiative heat exchange between surfaces in a transparent medium. Thus, if $\varepsilon'' < 0.05$, the use of formulas (20) and (21) is justified. For carbon dioxide and water vapor these values of ε'' correspond to $pl \approx (0.02-0.05) \cdot 10^5$ N/m ($0.02-0.05$ m $\cdot$ atm). In radiative heat-exchangers where one of the fluids is furnace gas the effective ray length is small in many cases and the partial pressure of CO_2 and H_2O does not exceed $0.3 \cdot 10^5$ N/m^2 and, hence, the value of ε'' does not exceed the indicated limit.

Besides this circumstance, however, it should be borne in mind (as McAdams [11] noted) that in the case of radiative heat exchange between surfaces separated by an emitting gas there is, on one hand, an increase in the temperature of the adiabatic envelope, which in our case is the casing of the apparatus and, on the other, an increase in the heat flux between the surfaces through "windows" in the absorption spectrum of the gases. The resultant effect is some screening due to the gas interlayer. Jakob [15] showed for a specific example of heat exchange between surfaces with $\vartheta = 1367°$ K, $\vartheta''_{12} = 811°$ K and separated by a gas layer with $\varepsilon_{H_2O} = 0.205$ that the presence of this layer reduces the amount of transmitted heat by only 8% in comparison with the case of absence of a gas between the surfaces.

We can conclude from the above that the recommendations made in this paper for the calculation of radiative-convective recuperators can also be used in cases where the hotter fluid is itself an emitting medium.

It should be noted in conclusion that the obtained approximate design relationships are also valid for the case where the colder gas flows through the intertube space. In this case, however, T'/ϑ^* must be > 0.5, since for lower values of this ratio the error in using formula (14) may be more than 10%.

NOTATION

T—temperature of heat-transfer fluids, °K; ϑ—temperature of partition or casing, °K; W—water equivalent of fluids, W/deg; α—heat transfer coefficient, W/m$^2\cdot$deg; s—mean heat-transfer perimeter, m; Bi—see (8); ε—emissivity; λ—thermal conductivity, W/m$\cdot$deg; δ—thickness of partition, m; Θ—dimensionless temperature, see (22); $R_{12} = 1/R_{21} = W_1/W_2$; ξ—see (18); F—heat-transfer surface, m^2; k—heat-transfer coefficient, W/m$^2\cdot$deg; x and z—coordinates, m. The subscripts 1 and 2, and also primes and double primes, refer, respectively, to the colder and hotter fluids; f refers to the outflow of fluid from the apparatus; x—coordinate along the length of the apparatus; the subscript 12 refers to the partition.

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